# Solving the QCD Hamiltonian for bound states

Elena Gubankova, Chueng-Ryong Ji, and Stephen R. Cotanch

Department of Physics, North Carolina State University, Raleigh, NC 27696-8202

Abstract. The systematic approach to study bound states in quantum chromodynamics is presented. The method utilizes nonperturbative flow equations in the confining background, that makes possible to perform perturbative renormalization and to bring the QCD Hamiltonian to a block-diagonal form with the number of quasiparticles conserving in each block. The effective block-diagonal Hamiltonian provides constituent description for hadron observables. The renormalized to the second order effective Hamiltonian of gluodynamics in the Coulomb gauge is obtained at low energies. The masses for scalar and pseudoscalar glueballs are predicted.

# I INTRODUCTION

One of the most difficult and less understood problems in quantum chromodynamics is the treatment of the bound state systems. There are different sources of difficulties. For example, it is quite a common observation known in the spectroscopy, that the splitting between the vector mesons does not depend on flavor, say

$$m(\rho') - m(\rho) \sim m(\psi') - m(J/\psi) \tag{1}$$

which is true experimentally. This fact can not be explained in terms of the canonical QCD interaction, which is given essentially by the strong coupling constant. Indeed, if expressed in terms of an invariant mass s, Eq.(1) implies that the  $J/\psi$  is dual to a much larger interval of s than the  $\rho$  because the c-quark is heavy. However, the coupling constant runs as a function of an invariant mass,  $\alpha(s)$ , and is flavor blind, thus the canonical QCD interaction should be much weaker for the  $J/\psi$  than for the  $\rho$ . This suggests that something is missing when described only in terms of the perturbation theory. The strong coupling constant alone does not provide for strong interactions being strong.

Consider the scaling of Quantum Chromodynamics from high to low energies. In the ultraviolet region (at the bare cutoff scale  $\Lambda \to \infty$ ) the strong interactions are given by canonical QCD, which is conformally invariant, in particular this means scaling invariance (there is no scale in the theory) in the chiral limit. Moreover

a perturbative treatment is possible due to asymptotic freedom. In asymptotic free theories (QCD) the coupling constant grows at low-energies and gets strong, that stops the asymptotic freedom at some moderate scale  $\Lambda_0 \sim \Lambda_{QCD}$ . This scale appears in the theory when the perturbative renormalization of the coupling constant is performed, and the  $\Lambda_0$  is the Landau pole in the effective running coupling constant provided the renormalization group invariance. This is called dimensional transmutation, when scaling invariance breaks through the renormalization. Then experiment tells us that this is not the only scale in the theory. There are at least two more characteristic scales in the hadron physics. The mass gap of the hadron bound state, say, given by the square root of the string tension, where the nonperturbative phenomenon of confinement takes place and the bound states of quarks and gluons form. The scalar  $\pi$ -meson sets up a scale of chiral symmetry breaking - another phenomenon of nonperturbative physics. The scales are displayed as  $m(\pi) \ll \sqrt{\sigma} \ll \Lambda_{QCD}$ .

To summarize: Knowing perturbation theory alone is not enough to describe bound states in hadronic physics.

Other difficulties in the QCD bound state problem are of a more general kind relativistic nature of quantum field theory. First, the number of particles in any state is not fixed because of particle creation and annihilation in vacuum. Second, there are states with (infinitely) large energies. An attempt to treat both problems was done in [3] by using the method of flow equations. The idea is to find a unitary transformation that transforms the Hamiltonian operator to a block-diagonal form, where each block conserves the number of particles. The Hamiltonian matrix can be represented in the particle number space as

$$H = \begin{pmatrix} PHP & PHQ \\ QHP & QHQ \end{pmatrix} \tag{2}$$

where P and Q are projection operators on the subspaces with different particle number content. The flow equations [1] bring the Hamiltonian matrix Eq.(2) to the form

$$H_{\text{eff}} = \begin{pmatrix} PH_{\text{eff}}P & 0\\ 0 & QH_{\text{eff}}Q \end{pmatrix} \tag{3}$$

where the two blocks of the effective Hamiltonian decouple from each other. It may be simpler then to solve for bound states within one block, say  $PH_{eff}P$ , than to diagonalize the complete Hamiltonian, Eq.(2), of the original problem. Since, generally, the number of particles in P and Q spaces is arbitrary, one can reduce in this way the bound state problem with many particles to a few body problem.

It turns out that the second question on possible presence of the ultraviolet divergences is solved also by the method of flow equations. Flow equations perform a set of (infinitesimal small) unitary transformations, where the flow parameter, which controls the transformation, has the dimension of the inverse of the energy square,

 $l \sim 1/E^2$ . Therefore by using flow equations to block-diagonalize the Hamiltonian one eliminates the particle number changing contributions not in one step but rather continuous for the different energy differences in sequence. This procedure enables one to separate the ultraviolet divergent contributions and to find the counterterms associated with these divergences. This covers the UV-renormalization for Hamiltonians [2].

To summarize: More generally, flow equations perform Hamiltonian renormalization in the "particle number" and in the "energy" spaces in the sense that the effects of the high Fock states and the effects of the large energies, respectively, are encoded in the effective low-energy Hamiltonian, which operates in the space of the few low Fock components.

This program was applied quite successfully to QED to calculate the positronium spectrum [3]. The key is the validity of the perturbation theory in the bare coupling constant for the characteristic energy scale of positronium bound state. Obviously, it is a bad idea to apply naively the same scheme for QCD. It is not possible to find the fixed number representation for the Hamiltonian in the case of *strong* interactions, where one does not have any control over the process of the creation and the annihilation in vacuum of bare quarks and gluons with *small* current masses. In the language of flow equations convergence can not be achieved when calculated in terms of bare parameters.

The way out is suggested by nature itself. One should consider "confined QCD". By using flow equations one constructs then the effective QCD Hamiltonian  $H_{eff}(q,g)$ , where current quarks and gluons acquire masses of the order of  $m_{constituent} \sim 1 GeV$  and become constituent degrees of freedom (see below). The value of the constituent mass plays the role of the energy gap between the sectors in  $H_{eff}$ . Schematically, the block-diagonal effective Hamiltonian has the form

$$H_{\text{eff}} = \begin{pmatrix} \frac{q\bar{q}}{|q\bar{q}g|} \\ \hline |q\bar{q}q\bar{q}| \\ |gg| \end{pmatrix}$$
 (4)

where q and g are constituent quarks and gluons, respectively; empty cells denote zero, to the order calculations are done, matrix elements. To this order the different sectors of the effective Hamiltonian, Eq.(4), describe approximately, when going down, the bound states of mesons, hybrids, glueballs. Actually, such a description with the fixed number of constituents is quasiclassical and nonrelativistic, and is known from the constituent quark model. Physically, the picture is the following. The strong confining interaction, acting inside each "diagonal" (particle number conserving) sector, produces heavy gluon (quark) only in the small volume – in the "bag". No free propagating heavy gluons (quarks) are produced. Therefore the "bags" do not interact with each other, decoupling in  $H_{eff}$  and approximating the hadron bound states.

The matrix elements of the "off-diagonal" (particle number changing) sectors are governed by the canonical interaction – typically by the Coulomb term of the strength equal to the inverse of Bohr radius or the current quark mass,  $m_{current}$  is of the order of several MeV. In the presence of the strong confining interaction in the "diagonal" sectors the mixing between the sectors is strongly suppressed. One can introduce a small parameter, say,

$$\alpha_s \frac{V_{12}}{E_1 - E_2} \sim 1 \cdot \frac{m_{current}}{m_{constituent}} \sim 0.1 - 0.01, \tag{5}$$

where  $V_{12}$  is the Coulomb interaction, and  $E_1 - E_2$  is the energy difference between the first and the second "diagonal" sectors. Perturbation theory with respect to the small parameter, Eq.(5), holds between the sectors (but not inside the sector where the confining interaction is strong). By applying flow equations to blockdiagonalize the Hamiltonian one gets to leading order a closed chain of decoupled equations, which can be solved analytically. The whole is true provided there is a strong confining interaction in the "diagonal" sectors.

To summarize: In the theory of strong interactions confinement is important to provide the bound states. In the present approach confinement makes it possible to bring, by flow equations, the QCD Hamiltonian to a block-diagonal form with a fixed number of quasiparticles in each sector. The elementary degrees of freedom (quasiparticles) become constituent quarks and gluons, which acquire masses of order 1 GeV. The block-diagonal effective Hamiltonian approximately describes then the different hadronic bound states.

The main idea of the approach is to find the representation for QCD Hamiltonian with the fixed number of quasiparticles, where the sectors with different particle number content decouple from each other. There can be some special cases when one should take into account the mixing between the sectors. In other words the physical state is not given by the pure component of the composite system. The mixing between the high excited state from the previous sector and the ground state from the next sector of the effective Hamiltonian may be possible (for example, the mixing between some excited meson and the low lying hybrid state). In systems with light quarks the influence of coupled channels can be essential. In the strongly coupled effective meson models one includes the effects from the coupled channels directly by mixing the scalar and pseudoscalar channels  $(q\bar{q}$  and  $q\bar{q}q\bar{q})$ . The effect is about 50 percent.

Special consideration is required in the case of the light quarks, where chiral symmetry breaking (CSB) is important. The present approach includes confinement and is like the "bag model" or the "constituent quark model", but it does not include CSB. By implementing CSB in this picture, the scalar  $\pi$ -meson can be viewed as a bound state of the two constituent quarks and simultaneously manifests the Goldstone nature.

Motivation: In order to disentangle the both problems of confinement and CSB we consider the pure gluodynamics (see the next section) [5]. The motivation for

this study is to set up a kind of a constituent gluon model, with the confining interaction imposed, to describe glueball bound states.

The specific difficulty of QCD is that the canonical QCD Lagrangian does not manifest explicitly confinement. As far as the mechanism of confinement is concerned one can proceed along several ways. To reveal confinement one uses the suitable formulation of QCD: lattice form, or the special choice of the gauge fixing (for example, maximal Abelian projection). Another option is to study other than QCD theories, but that have the same infrared behavior as QCD and are confined: Super Yang Mills theory, some toy gauge models (for example, Abelian Higgs Model). There may appear some unphysical degrees of freedom in these theories. If one is not interested in the mechanism of confinement, one includes the latter explicitly into QCD. The simplest way is to use the potential model, successfully tested in phenomenology, where the potential between the color charges is given by a sum of Coulomb and confining interactions. This suggest the definite choice of the gauge for the Hamiltonian. We work in the Coulomb gauge, where the Coulomb interaction arises from the gauge fixing procedure. We add then confinement to be able to block-diagonalize to the effective Hamiltonian, which describes the bound states.

The Coulomb gauge is the natural gauge to get the constituent hadron picture. The Coulomb interaction appears there not as a perturbative (propagating in time) one gluon exchange, but rather as a solution of the gauge fixing constrains. Therefore the Coulomb term describes an instantaneous interaction, which is consistent with the nonpropagating massive gluon arising in our approach. Note, that the massive gluon mode arises only in the presence of confinement and confinement sets the scale for the gluon mass. The obvious drawback of the approach is the violation of the gauge invariance by the massive gauge fields.

To summarize: The Coulomb gauge has an appealing property of the simple extension of the model to the confining case and is consistent with the constituent picture for hadrons.

# II LOW-ENERGY GLUODYNAMICS IN THE COULOMB GAUGE

As noted above we applied the method to pure gluodynamics [5]. In this section we outline the strategy of this study.

1.  $QCD H_{can}$ .

The starting point is the canonical QCD Hamiltonian (pure gluodynamics) in the Coulomb gauge  $\nabla \cdot \mathbf{A} = 0$ :  $H_{can}(\mathbf{A}, \mathbf{\Pi})$ , where physical degrees of freedom are the transverse gauge fields  $\mathbf{A}$  and their conjugate transverse momenta  $\mathbf{\Pi}$ .

2.  $H_{can} = H(g = 0) + O(g) + O(g^2)$ .

We expand the canonical QCD Hamiltonian in the Coulomb gauge perturbatively to the second order in the bare coupling constant. Then to the leading order the Faddeev-Popov determinant can be approximated by unity, that reduces the instantaneous term, arising from the gauge fixing, to the pure Coulomb interaction.

3. Current (perturbative) basis and the trivial vacuum  $|0\rangle$ .

We choose the trivial (perturbative) vacuum  $|0\rangle$  and construct the perturbative basis of free (*current*) particles:  $a^{\dagger}(\mathbf{k})|0\rangle$  creates one (perturbative) gluon with zero mass, i.e. the gluon energy  $\omega_{\mathbf{k}} = |\mathbf{k}|$ , etc., and the vacuum is defined as  $a|0\rangle = 0$ .

We express the canonical QCD Hamiltonian (section 2) in this perturbative Fock space, and normal order the result with respect to the trivial vacuum state  $|0\rangle$ . Denote the normal ordered canonical QCD Hamiltonian as  $:H_{can}:$ .

4. Regularization and perturbative renormalization (scheme).

The normal ordered Hamiltonian : $H_{can}$ : contains ultraviolet (UV) divergent terms (UV-divergent loop integrals). We regulate UV-divergences by the cutoff function  $f(q,\Lambda)$  (the explicit form of the cutoff function is specified further). This is the first time when we have introduced an energy scale in the theory – the bare cutoff  $\Lambda \to \infty$ . To remove the cutoff sensitivity we renormalize the Hamiltonian by adding the counterterms associated with these divergences. Schematically, the renormalized Hamiltonian is written as  $H_{ren}(\Lambda) = H_{can} + \delta X_{CT}(\Lambda)$ , where  $\delta X_{CT}(\Lambda)$  is a set of (unknown) counterterm operators, which we define further <sup>1</sup>.

5. Flow equations perturbatively.

To find the explicit form of the counterterms and to scale down the Hamiltonian we run flow equations perturbatively. Also the form of the cutoff (regulating) function is specified by flow equations. Generally, flow equations define the prescription of regularization and make possible to perform the perturbative renormalization<sup>2</sup>. Technically, since the Hamiltonian depends on the cutoff scale through the flow parameter, one finds in the given order of perturbation theory (PT) the divergent part of the difference between the Hamiltonian operators given at the two scales, say  $(H(\Lambda_2) - H(\Lambda_1))$  with  $\Lambda_{QCD} \ll \Lambda_2 \leq \Lambda_1 \leq \Lambda$ . One absorbs then these divergences in the counterterms – local operators with the symmetries of the canonical Hamiltonian, to provide the renormalization group invariance (called in the context of Hamiltonian renormalization "coupling coherence" [2]). This completes the procedure of renormalization, performed by flow equations, to this order. One can proceed in this way order by order in PT to find (all) the counterterms systematically. Note, that it is enough to find the gradient of the Hamiltonian in the energy space to define the counterterms. Renormalization group invariance (RGI) insures, that the renormalized Hamiltonian preserves the form of the (original) canonical Hamiltonian, but only the coupling constants and the mass operators (that are usually classified as relevant and marginal operators in renormalization group sense) start to run with the cutoff scale. (We do not consider here, at least

<sup>&</sup>lt;sup>1)</sup> In the given (perturbative) basis this equation reads  $:H_{ren}(\Lambda):=:H_{can}:+:\delta X_{CT}(\Lambda):$ , where ":" stands for normal ordering in the (perturbative) vacuum

<sup>&</sup>lt;sup>2)</sup> Flow equations perform a set of unitary transformations to block-diagonalize the Hamiltonian  $H(l, l_0) = U^{-1}(l, l_0)H(l_0)U(l, l_0)$ , where l is the flow parameter with the connection to the energy scale  $l = 1/\lambda^2$ ,  $l_0$  is the initial value corresponding to the bare cutoff  $\Lambda$  introduced by the regularization before (section 4).

in the few lowest orders of PT, possible irrelevant operators, that may cause new type of divergences than are carried by coupling constants and masses).

Using flow equations we run the effective Hamiltonian downwards from the bare cutoff  $\Lambda$  to some intermediate scale  $\Lambda_0 \sim \Lambda_{QCD}$ , where perturbation theory breaks down. Due to the RGI the "physical gluon" stays massless through this perturbative scaling. We can not proceed with flow equations perturbatively further. The result of this stage is the renormalized (to the second order of PT [5]), effective Hamiltonian, defined at some compositness scale  $\Lambda_0$ :  $:H_{ren}(\Lambda,\Lambda_0)$ : with  $\Lambda_0 \sim \Lambda_{QCD}$  and bare cutoff  $\Lambda \to \infty$ , and semicolon means normal-ordering in the trivial vacuum  $|0\rangle$ . Though the renormalized Hamiltonian is obtained in the perturbative frame, it can be represented (regardless of the Fock basis) in terms of the fields  $\mathbf{A}$  and  $\mathbf{\Pi}$  [5]. (It is a consequence of the RGI). We denote the resulting renormalized Hamiltonian at the scale  $\Lambda_0$  as  $H_{ren}(\Lambda,\Lambda_0)$ .

## 6. Confinement.

We introduce confinement as a linear rising potential, that enables to run flow equations "nonperturbatively" (see introduction) until complete diagonalization of the Hamiltonian. In the renormalization group sense this "spoils" the theory: there arises the massive gluon mode. But the presence of confinement is necessary to find the representation with a fixed number of quasiparticles (constituent massive gluons) for the effective Hamiltonian, which provides the constituent picture (see introduction). Confinement (string tension) sets the (hadron) scale for the gluon mass.

The instantaneous interaction contains two pieces, the sum of the Coulomb and confining potentials. Denote the renormalized effective Hamiltonian with confinement embedded as  $H_{eff}(\Lambda, \Lambda_0)$ .

7. Constituent (nonperturbative) basis and the QCD vacuum  $|\Omega\rangle$ .

As far as confinement is introduced the trivial vacuum  $|0\rangle$  and the perturbative basis of free (current) particles,  $\omega_{\mathbf{k}} = |\mathbf{k}|$ , define no longer the minimum ground state. Therefore, we introduce the (arbitrary) basis, where the gluon energy  $\omega_{\mathbf{k}}$  is kept unknown, and is defined further variationally. Correspondingly, the (nontrivial) QCD vacuum  $|\Omega\rangle$  is defined as  $\alpha|\Omega\rangle = 0$ , and the Fock space of constituent particles is given:  $\alpha^{\dagger}|\Omega\rangle$  creates the quasiparticle with the energy  $\omega_{\mathbf{k}}$ , etc. <sup>3</sup>. The renormalized effective Hamiltonian  $H_{eff}(\Lambda, \Lambda_0)$  at the scale  $\Lambda_0$  (section 6), written through the physical fields  $\mathbf{A}$  and  $\mathbf{\Pi}$  and having confinement, is decomposed in the trial (constituent) basis and normal-ordered with respect to QCD vacuum  $|\Omega\rangle$ . The unknown gluon energy is variational parameter in the calculations. We combine the terms in the effective Hamiltonian in each particle number sector according to the

<sup>3)</sup> The change of basis from the (perturbative) current,  $\omega_{\mathbf{k}} = |\mathbf{k}|$ , to the (nonperturbative) constituent, with some  $\omega_{\mathbf{k}}$ , can be written as Bogoluibov-Valatin (BV) transformation from the "old",  $a, a^{\dagger}$ , to the "new",  $\alpha, \alpha^{\dagger}$ , operators:  $a_{\mathbf{k}} = \mathrm{ch}\phi_{\mathbf{k}}\alpha_{\mathbf{k}} + \mathrm{sh}\phi_{\mathbf{k}}\alpha_{-\mathbf{k}}^{\dagger}$  with BV angle  $\phi_{\mathbf{k}}$  given by  $\mathrm{ch}\phi_{\mathbf{k}} = 1/2(\sqrt{k/\omega_{\mathbf{k}}} + \sqrt{\omega_{\mathbf{k}}/k})$ . The connection between the "old",  $|0\rangle$ , and the "new",  $|\Omega\rangle$ , vacuum states is given  $|\Omega\rangle = \exp\left(\frac{1}{2}\sum_k \mathrm{th}\phi_{\mathbf{k}}a_{-\mathbf{k}}^{\dagger}a_{-\mathbf{k}}^{\dagger}\right)|0\rangle$ . It was used in the work [4] to transform the QCD Hamiltonian into the constituent basis.

power of coupling constant  $O(g^n)$   $(n = 0, 1, 2)^4$ . In the absence of confinement the effective Hamiltonian preserves the form of canonical Hamiltonian due to RGI, with the proper change  $|\mathbf{k}| \to \omega_{\mathbf{k}}$ . In the presence of confinement the canonical form is violated by the second order terms in the effective Hamiltonian, which contribute higher orders  $O(g^3)$ , etc. in flow equations.

We aim to find the effective Hamiltonian after the scaling downwards from  $\Lambda_0$  to a hadron scale, say  $\sqrt{\sigma}$ . Since the effective Hamiltonian preserves the canonical form at least to the second order, the "perturbative" terms obtained by flow equations in section 5 match the "nonperturbative" terms arising when applied flow equations to  $H_{eff}(\Lambda, \Lambda_0)$ .

We denote the effective Hamiltonian in constituent basis as :: $H_{eff}(\Lambda, \Lambda_0)$ ::, where "::" stands for normal-ordering in the QCD vacuum.

# 8. Flow equations in the confining background.

We run flow equations in the confining background to block-diagonalize the effective Hamiltonian  $:: H_{eff}(\Lambda, \Lambda_0)::$  in the nonperturbative basis and to find consistently all the terms to the second order. Free Hamiltonian and confining interaction are included in "diagonal" sector, the triple-gluon vertex forms "nondiagonal" sectors, that should be eliminated. We bring the Hamiltonian  $:: H_{eff}(\Lambda, \Lambda_0)::$  to a block-diagonal form, where diagonal blocks decouple from each other including the second order. The leading UV-behavior of the arising to the second order terms is cancelled by the mass counterterm. Generally, this approach allows to include perturbative QCD corrections into nonperturbative calculations of many-body techniques. The resulting block-diagonal effective, renormalized Hamiltonian is given at hadronic scale,  $:: H_{eff}(\Lambda, \sqrt{\sigma})::$  For simplicity we denote it as  $H_{eff}$ .

# 9. Gap equation (variational calculations).

The requirement of block-diagonal form does not fix the effective Hamiltonian completely. The remaining freedom to unitary transform inside of each block is fixed by minimizing the ground state (the vacuum expectation value of the effective Hamiltonian with respect to QCD vacuum)

$$d\langle \Omega | H_{eff} | \Omega \rangle / d\omega = 0 \tag{6}$$

to find the trial gluon energy  $\omega(\mathbf{k})$ . The variational function  $\omega(\mathbf{k})$  is defined. As a result the gluon acquires a nonzero mass,  $m \sim 0.5 GeV$ , and an iterative procedure of flow equations is performed with respect to the small parameter 1/m. The next following sector in the effective Hamiltonian is suppressed by this factor, that provides the convergence for the flow equations (see introduction).

# 10. Solving for $H_{eff}$ .

There are two parameters in the method, the two scales:  $\Lambda_0$ , which defines the counterterms and regulates the perturbative radiative corrections to the effective Hamiltonian  $H_{eff}$ , and  $\sqrt{\sigma}$ , where  $\sigma$  is the string tension defining the nonperturbative confining potential.

<sup>&</sup>lt;sup>4)</sup> The higher order terms in the effective Hamiltonian are suppressed by the inverse powers of (heavy) gluon mass, which is of order of hadron scale (see introduction).

Since the effective Hamiltonian is block-diagonal, one can solve for the bound states in any interesting sector (actually in the few lowest sectors). We solve for the glueball bound state in the two-body sector. The result is the glueball spectrum.

# III SUMMARY

### 1. Renormalization

Renormalization was performed to the second order. We combined the individual counterterms in one- and zero-body sectors to the resulting mass counterterm, written in the field representation (independent of the basis)  $\delta X_{CT}(\Lambda) = m^2 \text{Tr} \int d\mathbf{x} \mathbf{A}^2(\mathbf{x})$  with  $m^2 = -\frac{\alpha_s}{\pi} N_c \frac{11}{12} \Lambda^2$  Remarkably, when the quark sector is added in the same fashion, the algebraic coefficient in the propagator correction reproduces the QCD  $\beta$ -function. This particular feature of the Coulomb gauge supports our regularization prescription, which follows from flow equations.

#### 2. Glueball

We specify the two parameters: the string tension is defined by the lattice calculations  $\sigma = 0.2 GeV^2$ ; the cutoff  $\Lambda_0$  is found from the gluon condensate to agree with the result of the sum rules, the condensate term is obtained  $\langle G^2 \rangle \sim 1.3 \cdot 10^{-2} GeV^4$  with  $\Lambda_0 = 4 GeV$ .

The solution of the gap equation can be parameterized as  $\omega(\mathbf{k}) = k + m(0)\exp(-k/\kappa)$ , where the effective gluon mass is obtained m(0) = 0.90 GeV and  $\kappa = 0.95 GeV$ .

The glueball mass spectrum in scalar and pseudoscalar channels is given in Tamm-Dancoff approximation in [5]. Roughly, the mass of the lowest scalar glueball  $0^{++}$ , 1760MeV, is twice of the effective gluon mass m(0).

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